



Bootstrapping the Economy

Generating Consistent Scenarios for Risk Management

Risk Day 2006, ETH Zürich, Oct 20, 2006

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Economic Scenario Generator (ESG): Motivation, purpose

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- Consistent **stochastic scenarios** for the future of the economy:
 - Modeling invested **assets** and **liabilities** affected by economy
 - Results: expected returns, **risks**, full distributions
 - **Business decisions** (incl. asset allocation, hedging of risks)
 - Our application: Asset-Liability Mgmt (ALM) for insurance firms
- **Multi-purpose** ESG with strong requirement of realistic behavior:
 - **Many** economic variables: yield curves, asset classes, inflation, ...
 - 6 currency zones (flexible) and FX rates
 - Correlations, **dependencies** between all economic variables
 - **Heavy tails** of distributions
 - Realistic behavior of autoregressive **volatility** clusters
 - Realistic, arbitrage-free **yield-curve behavior**
 - Short-term and long-term scenarios (months → 40 years)

Economic Scenario Generator (ESG): Implementation, bootstrapping

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- Our implementation: Economic Scenario Generator (ESG) based on **bootstrapping**.
- Bootstrapping **historical** behaviors for simulating the **future**
- Bootstrapping is a method that automatically fulfills many requirements, e.g. realistic **dependencies** between variables.
- Some requirements need **additional** modeling:
 - **Tail correction** for modeling **heavy tails**
 - “**GARCH** filter” for autoregressive clustering of **volatility**
 - **Yield curve** preprocessing (using forward interest rates) in order to obtain arbitrage-free, realistic behaviors.
 - Weak **mean reversion** of some variables (long-term behavior).

Scenario generation methods: parametric or non-parametric ?

■ **Parametric ESG:**

- Econometric model with parameters for all variables
- Calibration of parameters using historical data
- Fitting of initial state is needed: initial values of driver variables
- Examples: Hull-White, most models used for option pricing

■ **Non-parametric ESG:**

- Direct algorithm for simulating the future
- Directly using historical data (not just through prior calibration)
- Initial variable values = natural simulation start (no fitting needed)
- Method presented here: **bootstrapping** (or **resampling**)

- Our bootstrapping method has some secondary elements with parameters: “**semi-parametric**”.

Economic Scenario Generator (ESG): economic variables and data

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- Key variables describe the economy: interest rates (yield curves), FX rates, inflation, GDP, equity indices, real estate + hedge fund indices
- Modeling the “world” economy: six **currency zones** (US, EU, GB, JP, CH, AU)
- **Derived variables** (such as values of bond portfolios or mortgage-backed security investments) are supported, for each scenario.
- We need **historical data** for these variables.
- Data source:
Data providers such as Bloomberg → database → import in the ESG
Historical data plus continuous updates over time

Economic variables: historical data vectors as a basis

	Data vector	31.12.2003	31.03.2004
economic variables ↑	USD Equity	3337.41	3384.48
	USD CPI	120.9	123.0
	USD GDP	11262	11448
	USD IR 3m	0.90%	0.92%
	USD IR 6m	0.99%	1.01%
	USD IR 1y	1.22%	1.11%
	USD IR
	USD IR 30y	5.01%	4.70%
	USD Hedge Funds	286.9	296.7
	EUR Equity	2498.85	2578.43
	EUR CPI	114.2	115.0
	EUR GDP	1584316	n.a.
	EUR IR 3m	2.04%	1.89%
	EUR IR 6m	2.06%	1.85%
	EUR IR 1y	2.18%	1.87%
	EUR IR
	EUR IR 30y	4.95%	4.78%
	EUR FX Rate	1.2591	1.2316
	GBP Equity	5577.71	5531.21
	GBP
	GBP FX Rate	1.7858	1.8463
	JPY Equity	1064.61	1190.33
	JPY
	JPY FX Rate	0.0093088	0.0095946
	AUD Equity	2617.79	2744.92
	AUD
	AUD FX Rate	0.75200	0.76695
	CHF Equity	1540.54	1589.48
	CHF
	CHF FX Rate	0.80694	0.79005
CHF Real Estate	176.55	178.14	

time →

■ Currencies:

- AUD, CHF, EUR, GBP, JPY, USD

■ Economic variables:

- Equity index (MSCI)
- Foreign exchange rate
- Inflation (CPI)
- Gross domestic product
- Risk-free yield curves
- Hedge funds index (USD)
- MBS index (USD, based on yield curves)
- Real Estate
- Economic indicator (derived, for business cycle of liabilities)

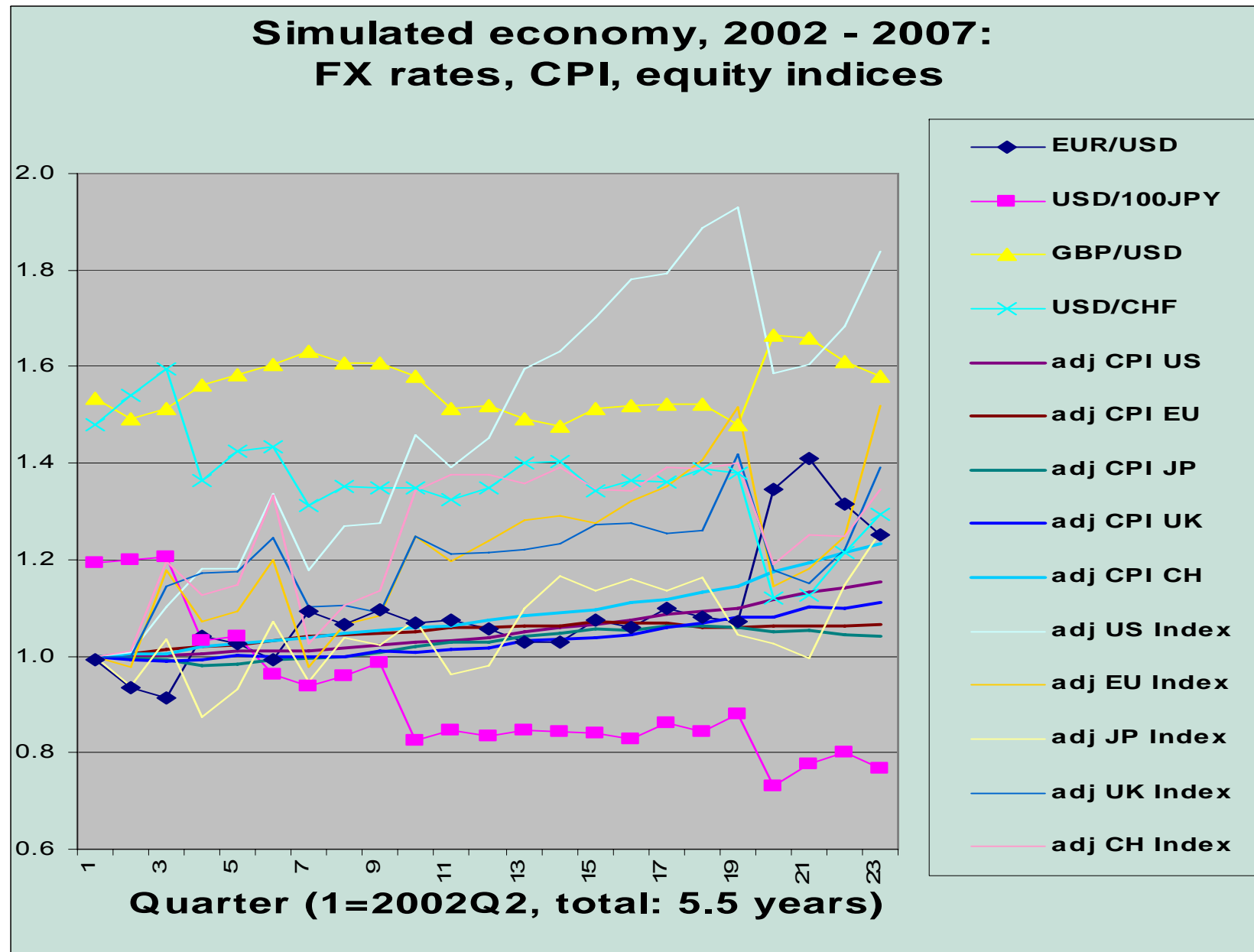
What does an ESG produce?

- An ESG produces scenarios of the future behavior all variables.
- There is a start date (e.g. today). All scenarios start from there.
- A scenario has future values for all the modeled variables.
- The presented ESG is based on quarterly time steps.
- Many different scenarios together define the space of likely future developments. We use **thousands** of scenarios!
- We compute expectation values = mean values of scenarios.
- We also compute **risks**, e.g. standard deviations or **expected shortfalls**: mean values of the 1% worst scenarios
- **Dependencies** between variables are reproduced.

Example of one economic scenario, bootstrapping, several variables

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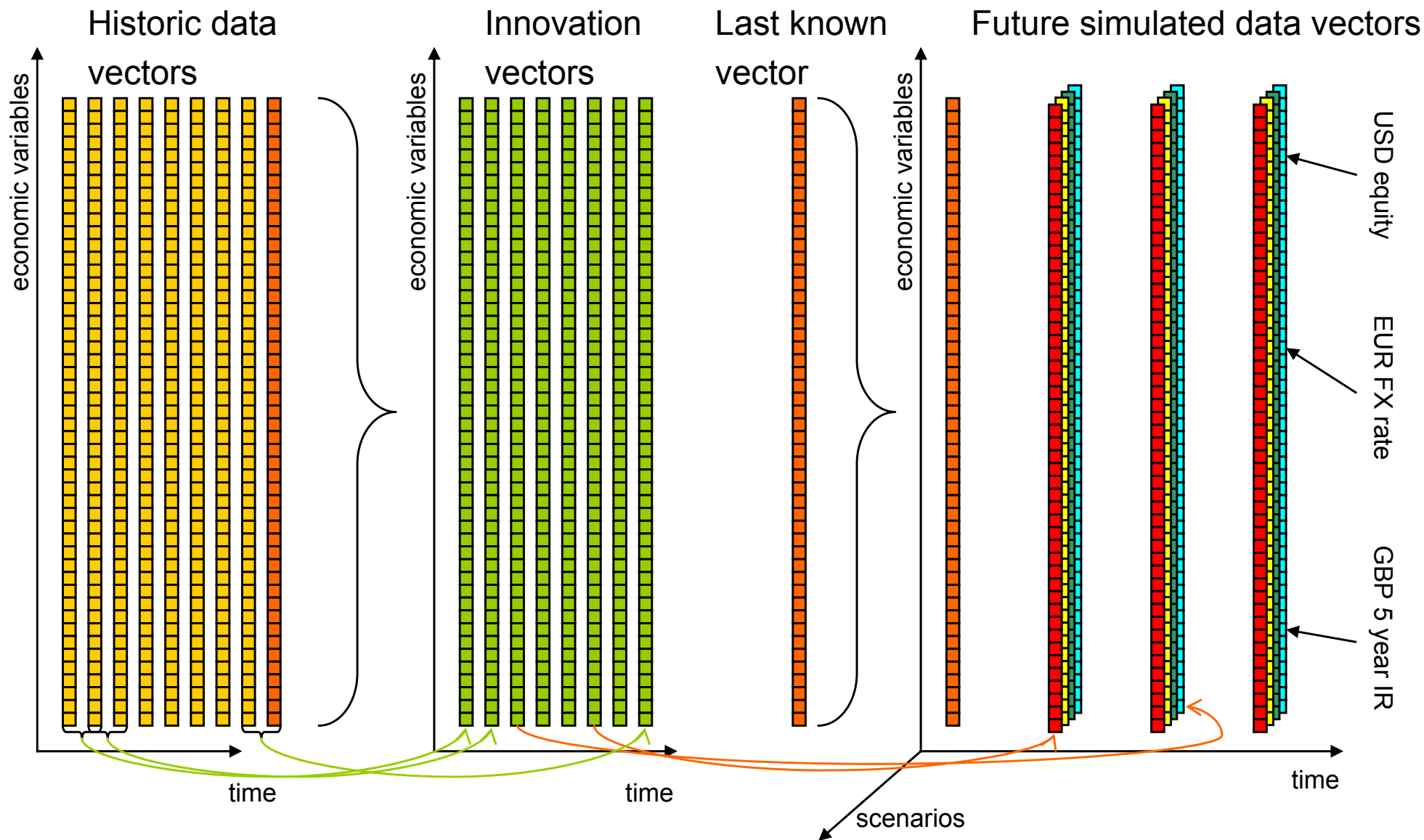
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The bootstrapping method: data sample, innovations, simulation

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The bootstrapping method: analyze history to obtain innovations

1. Start: Historical data sample, all variables
2. Transform variables: logarithmic; deseasonalizing inflation; interest rate case is complex. Goal: additivity, no limit for mapped variables.
3. Compute **market expectations** of all variables, for one time step ahead: forward rates, growth expectations, ...
4. Small corrections of these market expectations: long-term **mean reversion** (e.g. PPP), business cycle corrections, ...
5. Computation of historical **innovations**:
Innovation = New value - Prior expectation of that value
6. GARCH filter: GARCH model of innovations, use its **residuals** as the new innovations.
7. The innovations represent the element of **surprise** or **risk** in the economic development and will be used for bootstrapping.

The bootstrapping method: simulation of scenarios

1. Start a simulation from the last time point of historical data (and the market expectations made there).
2. The central bootstrapping step:
Randomly pick a **whole** historical **innovation vector**
→ historical dependencies between innovations are maintained.
3. Make a “tail correction”; rescale the innovations with GARCH.
4. Simulate a future time step:
Simulated value = Prior expectation of value + Innovation
5. Re-seasonalize inflation, re-transform economic variables to their original form.
6. Repeat the simulation step several times to obtain one full scenario.
7. Repeat this several times to obtain a sufficient number of scenarios.

Heavy tails in the ESG

- **Market shocks** and **extreme price moves** matter in economic risk assessment. Look at the **tails** of distributions!
- Bootstrapping covers **some** shocks: those contained in historical data.
- The size of historical samples (for many variables) is limited.
- Extreme shocks (such as a “1 in 200 years” event) are probably missing in the recorded history.
- Solution in the ESG: use “**tail-corrected**” innovations.
- **Corrected innovation = Historical innovation * η** ,
where η is a positive random variable with a mean square of 1 and a **Pareto-shaped upper tail** (with a realistic tail index).
- Due to this tail correction, some occasional simulation scenarios will behave like “**stress scenarios**”: larger shocks than in the samples.

Stochastic correction factor to obtain heavy-tailed innovation

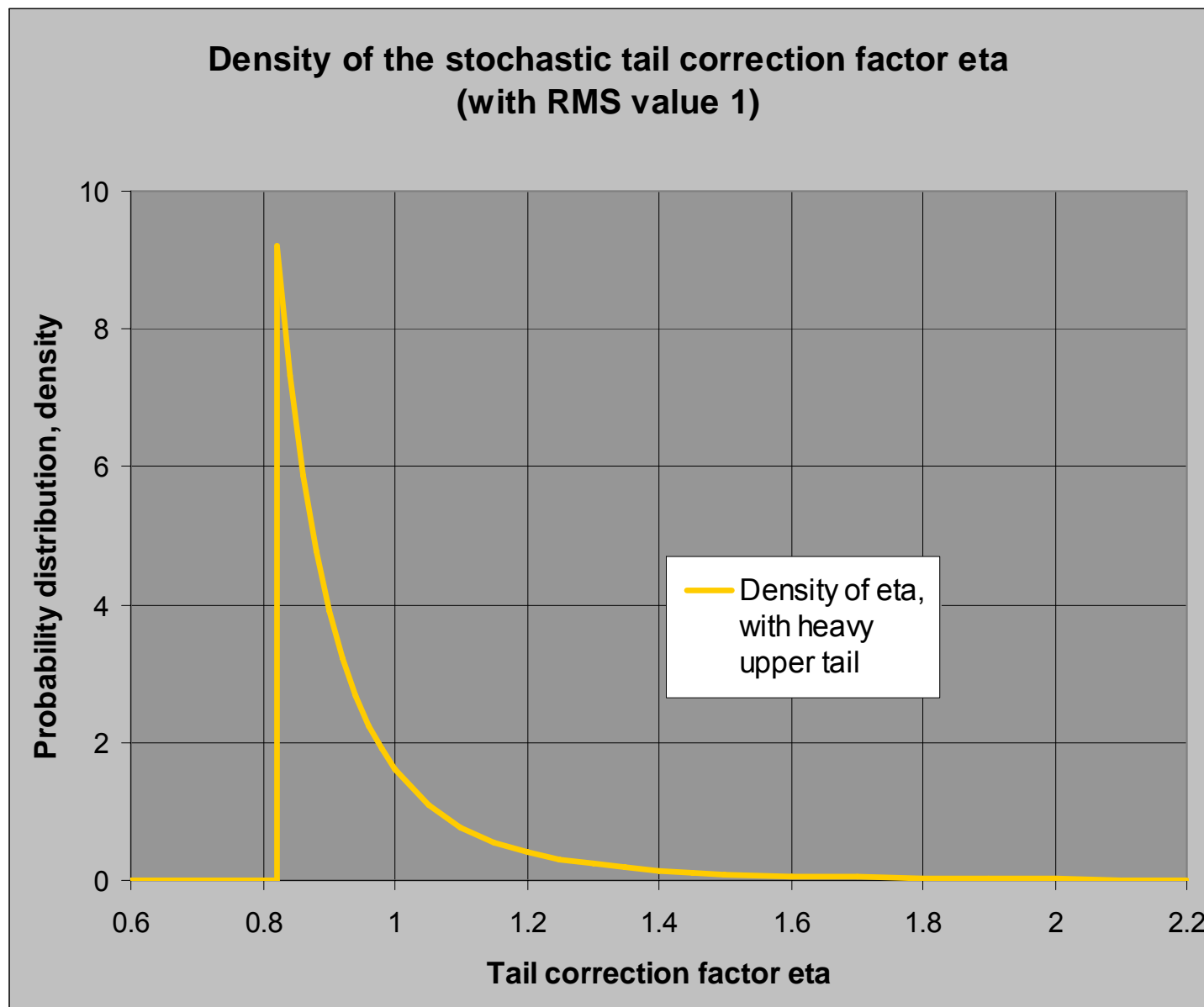
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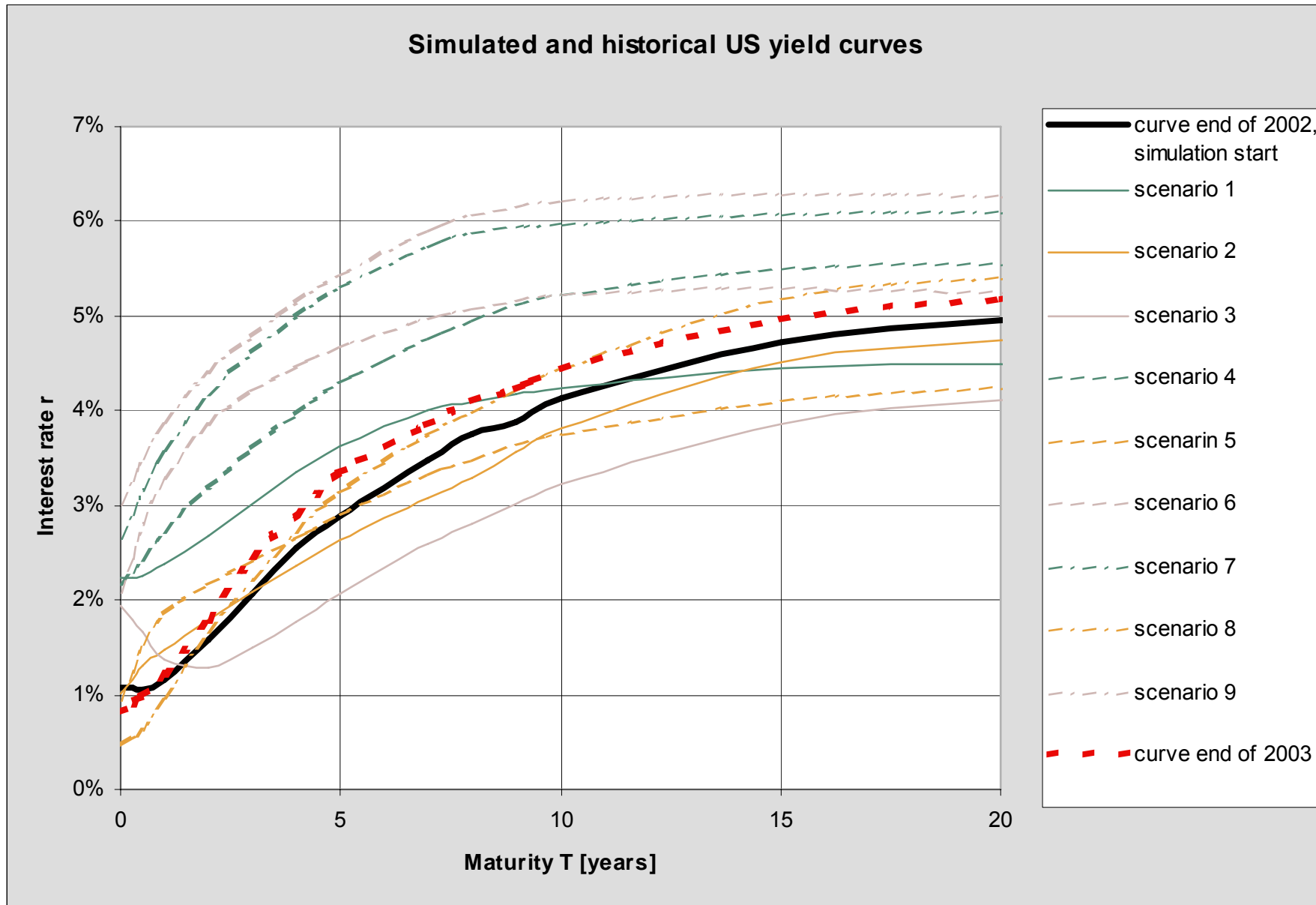


- Stochastic correction factor η to be applied to all bootstrapped innovations
- Root of mean square (RMS) = 1
→ corrected innovations have unchanged variance
- Heavy tail and other parameters configurable (see paper)

Yield curve modeling of the ESG

- The ESG includes a **complete yield curve model**, which is
 - arbitrage-free,
 - using **forward** interest rates as market expectation values,
 - modeling forward interest rates as economic variables, using the bootstrapping method,
 - accounting for the asymmetry of interest rates (**> 0**),
 - producing realistic, smooth forms of yield curves,
 - using the exact, current yield curve as simulation start,
 - allowing for **long-run** simulations (using weak mean reversion),
 - modeling yield curves of **many countries**, with dependencies.
- These properties → **multi-purpose** model: expectations, variances, correlations, **risk** assessment, **not only for derivatives pricing**.

Behavior of simulated yield curve, example: simulation of year 2003



Modeling the yield curve dynamics: some technical points

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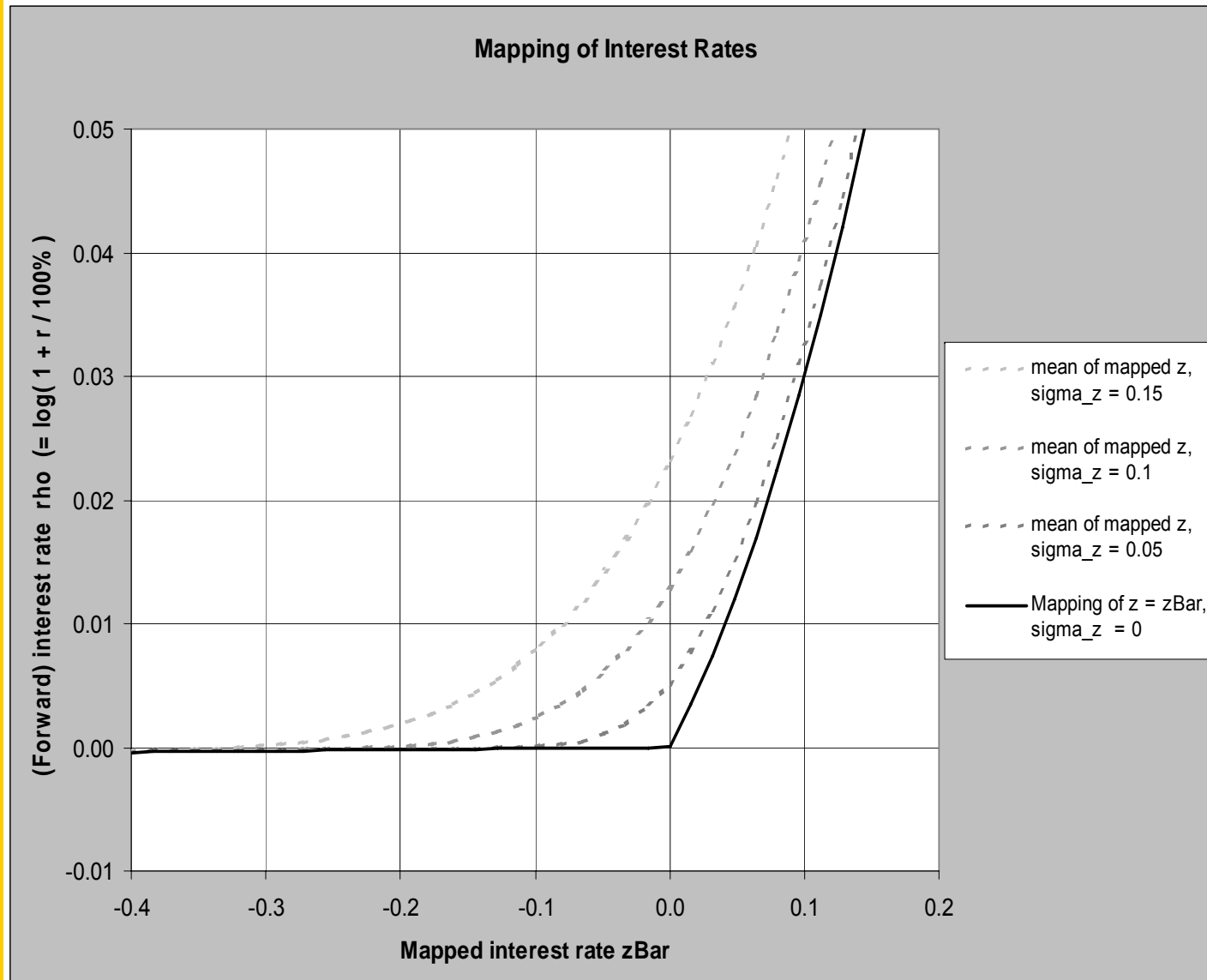
- Interest rates and yield curves follow a complex dynamic process – the toughest challenge for any economic scenario generation model.
- Model **forward** interest rates for fixed dates (as in futures markets).
- **Map** forward rates in a nonlinear way to **avoid interest rates < 0** .
- Account for **asymmetric** long-term **interest rate risk**: downside is limited, upside is unlimited.
- Holding long-term deposits is risky: “term premium”.
- This is why the standard form of yield curves is upward slanting.
- **Test** for any economic scenario generator:
Is the ESG able to produce the standard form as a long-run mean?

Mapping of forward interest rates, depending on maturity

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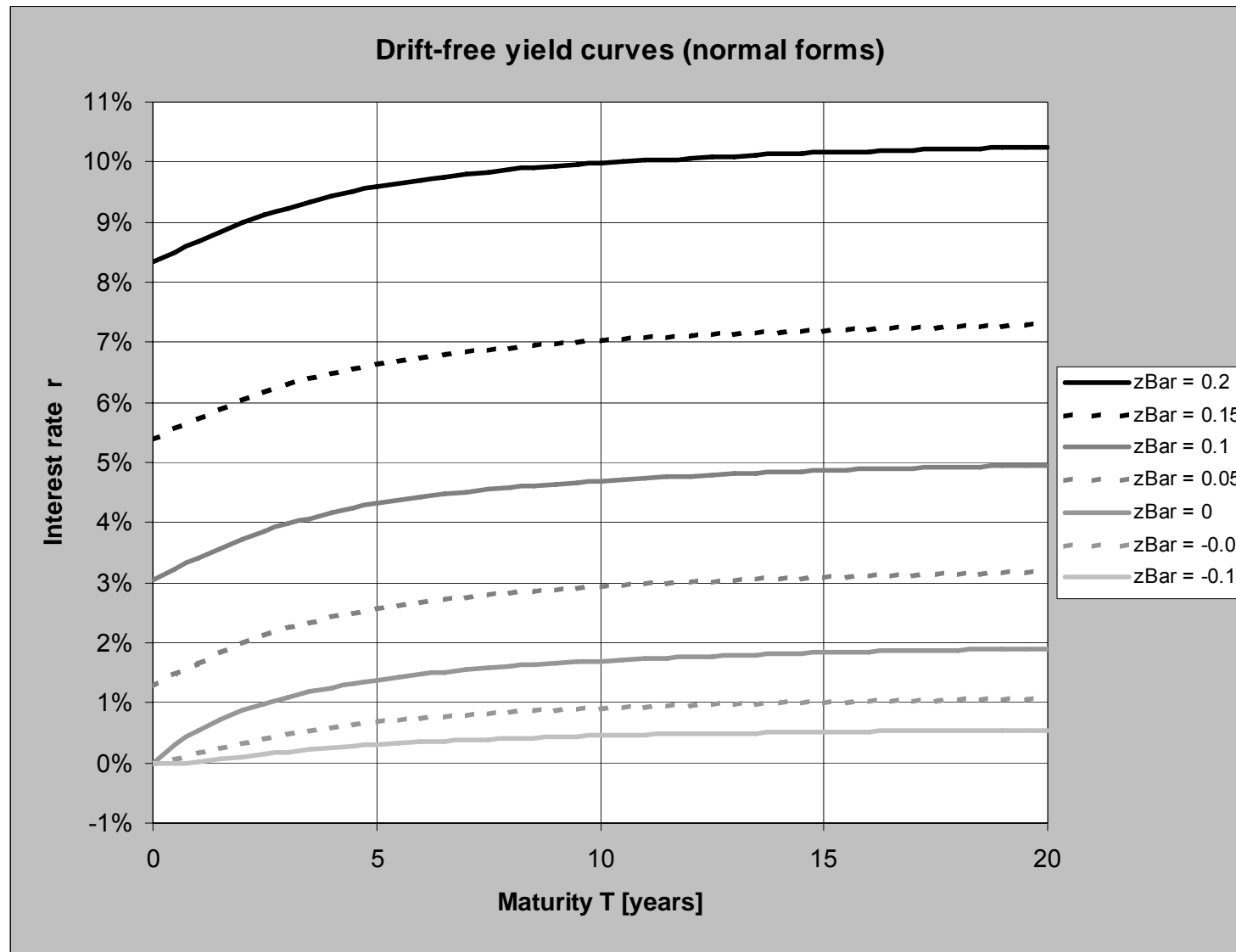


- **Forward IR** as a function of the **mapped IR** (depending on time to maturity)
- The mapped IR is **unlimited**, but forward IRs are constrained to ≥ 0 (mostly)
- The mapped IR is the modeled economic variable with **additive innovations** (bootstrapping).

Standard forms of ESG yield curves (in case of continued zero innovation)

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- What if innovations are zero?
- The ESG yield curves do not become flat; they adopt the “standard form”!
- Simulated yield curves (with nonzero innovations) “oscillate” around these standard forms.

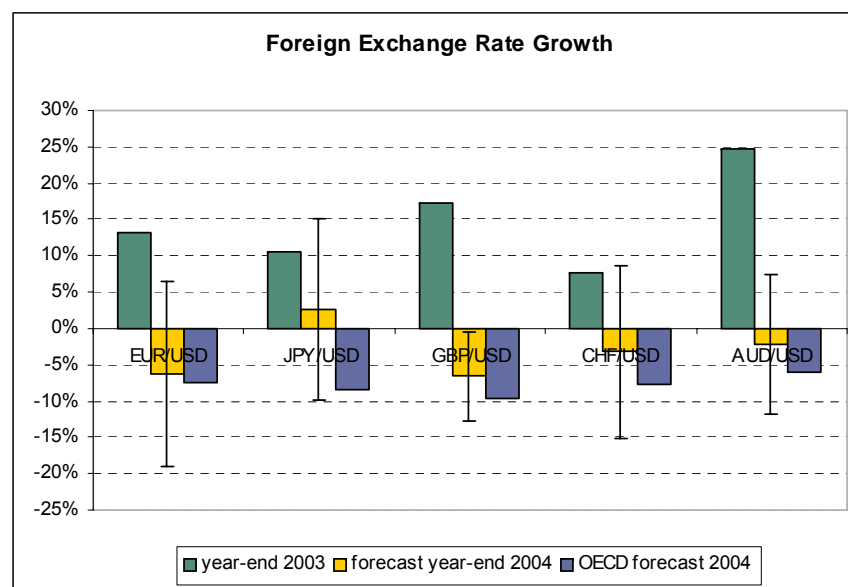
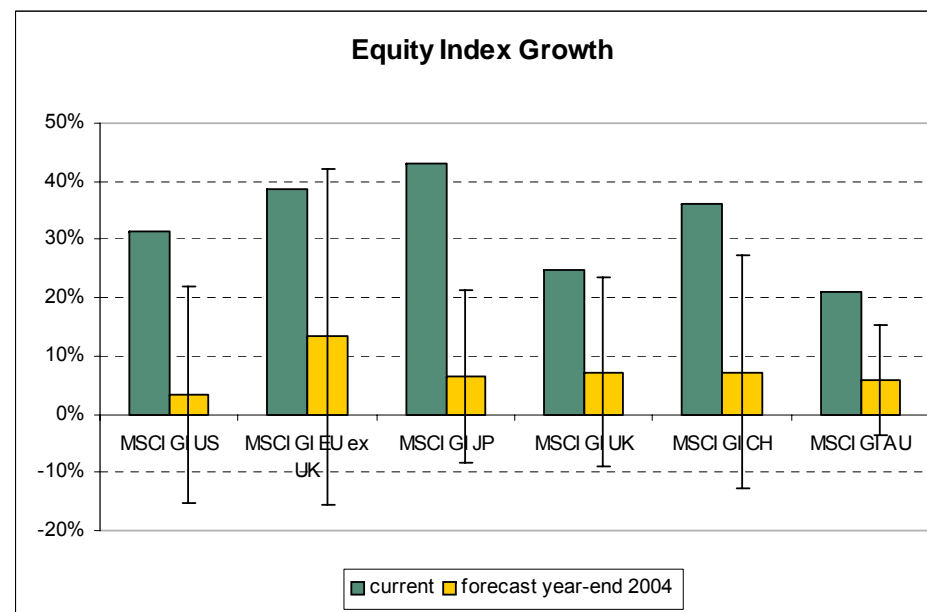
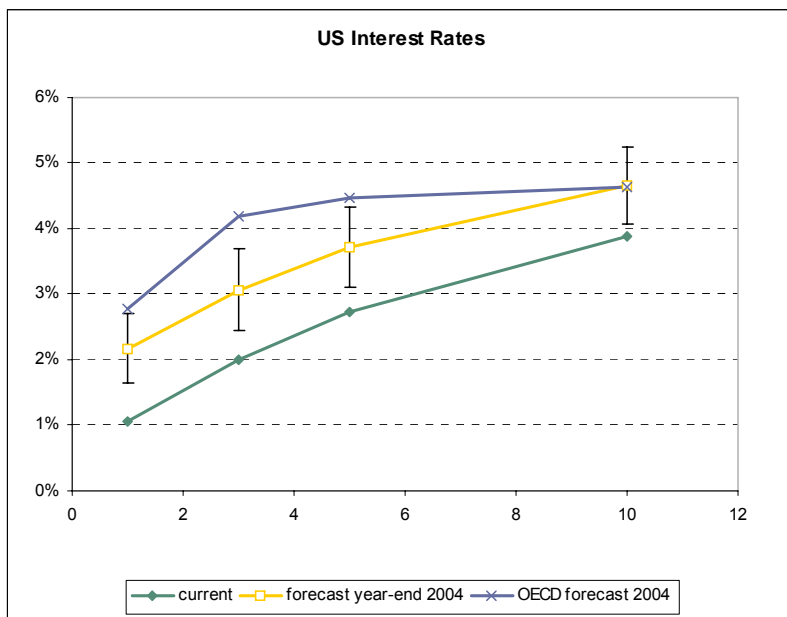
Testing the quality of ESG scenarios

- The ESG produces many scenarios, i.e. many different “forecasts”.
- Thousands of scenarios together define forecast **distributions**.
- **Backtesting**: How well did known variable values fit into their prior forecast distributions?
- Testing Method: **Probability Integral Transform (PIT)**, Diebold et al. (1998). Determine the **cumulative probabilities** of real variable values, given the distribution of their prior simulated values.
- This means mapping the distribution to a uniform distribution $U(0,1)$.
- The series of so obtained probabilities should be i.i.d.. Many tests!
- We may need additional tests (e.g. for the correct prediction of dependencies).

Example of results: economic forecasts (March 2004, 3 quarters ahead)

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Conclusions (1): Economic Scenario Generator

- The ESG based on bootstrapping has several **advantages**:
- **Wide coverage** of economic variables, **flexible** and modular when adding more variables (many results → need many quality tests).
- Automatic preservation of **dependencies** between variables.
- A full-fledged **yield-curve model** is at the heart of this ESG.
- Short-term simulations (with exact initial conditions) are possible as well as long-term simulations (where mean reversion plays a role).
- Treatment of **heavy tails** and **volatility clustering** (due to refinements of the method).
- The method is essentially **non-parametric** → no large calibration problems.
- The method was presented at scientific conferences, to be published.

Thank you ...

... for your attention!

Appendix:

Volatility modeling: GARCH

- The **volatility** of most variables in finance exhibits **autoregressive clusters**: long periods of low volatility / long periods of high volatility.
- The bootstrapping method (random sampling) **disrupts** those clusters.
- Solution: **GARCH** model to **re-introduce volatility clusters**:
 - GARCH model for the volatility σ_i of the time series of innovations x_i , for each variable, where $x_i \in N(0, \sigma_i^2)$
 - Iterative GARCH(1,1) equation: $\sigma_i^2 = \alpha_0 + \alpha x_{i-1} + \beta \sigma_{i-1}^2$
 - Robust calibration of the GARCH parameters on historical samples: α_0, α, β
- The bootstrapping method uses **normalized** innovations: x_i / σ_i .
- At each simulation step, the resampled innovation x_i / σ_i is **rescaled** by the current, updated GARCH volatility $\sigma_j \rightarrow$ new **innovation** $x_i \sigma_j / \sigma_i$

Appendix: GARCH, a widely used model: volatility with a memory

- Proposed and investigated by Engle (1982, Nobel Prize 2003) and Bollerslev (1986). Model for **volatility** σ of time series x .
GARCH: “Generalized Autoregressive Conditional Heteroskedasticity”
- Mostly used: GARCH(1,1), $x_i \in N(0, \sigma_i^2)$
 $\sigma_i^2 = \alpha_0 + \alpha x_{i-1} + \beta \sigma_{i-1}^2$ with three parameters: α_0, α, β
- **Stationary** GARCH(1,1) means: $\alpha + \beta < 1$
This is a condition for using GARCH(1,1) in simulations of the **future**. (Non-stationary GARCH processes would lead to “exploding drift”).
- Special case: IGARCH process with $\alpha + \beta = 1$
RiskMetrics is using IGARCH to compute **current volatility** (of recent price moves), but it is unusable for simulation into the future.
- **Calibration** of GARCH parameters through (Quasi-) **Maximum Likelihood**, observing (robust) stationarity: $\alpha + \beta < 1 - \varepsilon$

Appendix: Long-term scenarios: Mean reversion in the ESG

- In the long run, the bootstrapping method makes simulated variables behave like random walks with drift.
- This corresponds to the empirically observed behavior in most cases (equity indices, FX rates, ...).
- However, some variables exhibit weak, long-term **mean reversion**. Examples: interest rate levels, inflation, purchase-power parity.
- Numerical example: Interest rates are **unlikely** to drift up to 50% or down to -2%, even if the **simulation period is very long**.
- Mean reversion materializes as a small, reverting correction of the “*prior market expectation*”, mainly if current values are extreme.
- (Reminder: This matters in the application of ESG **innovations**:
Innovation = New value - *Prior expectation of value*)